Probability

Finite Math

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Empirical Approach

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In an empirical approach to probability, we run the experiment several times, and assign probabilities according to the frequency which with outcomes occurred. For example, if we flip a coin 1000 times and get 373 heads and 627 tails, we would be tempted to assign probabilities as

$$P(H) = \frac{373}{1000} \qquad P(T) = \frac{623}{1000}$$

since it reflects the results of an extensive experiment.

Empirical Approach

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Example

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Refer back to the example where we roll two dice. If we assume that every simple event is equally likely:

(a) *What is the probability of a simple event happening?*

(b) *What are the possible numbers that the two dice could add up to?*

(c) *What are the probability of each of the events in part (b) happening?*

Experiment Time!

In the previous example, we came up with the probabilities of rolling a given number as the sum of two dice. Now we will take an empirical approach to test this and see if those numbers hold up! Roll your pair of dice 50 times and record the sums of your rolls (I suggest making a table with the possible outcomes and just using tally marks). Compute the approximate empirical probabilities for the different sums of dice based off your experiment. How does it compare to the theoretical probabilities with the equally likely assumption?

Unions and Intersections

Definition (Union and Intersection of Events)

If A and B are two events in a sample space S (i.e., $A \subset S$ and $B \subset S$), we define two new events:

- *The event A or B is the union $A \cup B$*
- *The event A and B is the intersection $A \cap B$*

Example

Example

Suppose we are rolling a single fair die (each number is equally likely), so our sample set is

$$S = \{1, 2, 3, 4, 5, 6\}.$$

(a) *What is the probability of rolling a number which is even and divisible by 3?*

(b) *What is the probability of rolling a number which is even or divisible by 3?*

Now You Try It!

Example

Suppose we are rolling a single fair die, so our sample set is

$$S = \{1, 2, 3, 4, 5, 6\}.$$

(a) *What is the probability of rolling a number which is odd and prime?*

(b) *What is the probability of rolling a number which is odd or prime?*

Probability of a Union

Theorem (Probability of the Union of Two Events)

For any events A and B,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

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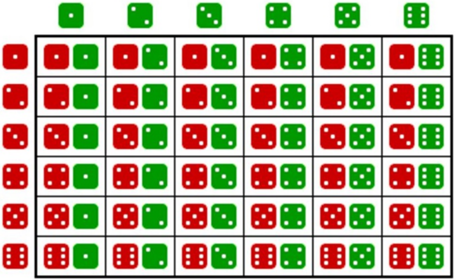
Remark

Note the similarity to the formula for the addition principle:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

Example



Example

Suppose that two fair dice are rolled.

(a) *What is the probability that a sum of 7 or 11 turns up?*

(b) *What is the probability that both dice turn up the same or that a sum less than 5 turns up?*

Example

Example

What is the probability that a number selected at random from the first 500 *positive integers is:*

(a) *divisible by* 3 *or* 4 *?*

(b) *divisible by* 4 *or* 6 *?*

Complements

Let $S = \{e_1, e_2, ..., e_n\}$ be a sample space and let E be some event. Then the set E' is also an event and since $E \cap E' = \varnothing$ and $E \cup E' = S$, then we have

$$P(S) = P(E \cup E') = P(E) + P(E') = 1.$$

So it follows that

$$P(E) = 1 - P(E') \qquad \textit{and} \qquad P(E') = 1 - P(E).$$

If E is an event, then E' is "the event that E *does not* happens."
As a simple example, if E is the event that it snows outside and $P(E) = .35$, then E' is the event that it does not snow and $P(E') = .65 = 1 - .35$.

Example

Example

A shipment of 45 *precision parts, including* 9 *that are defective, is sent to an assembly plant. The quality control division selects* 10 *at random for testing and rejects the entire shipment if* 1 *or more in the sample are found to be defective. What is the probability that the shipment will be rejected?*

Now You Try It!

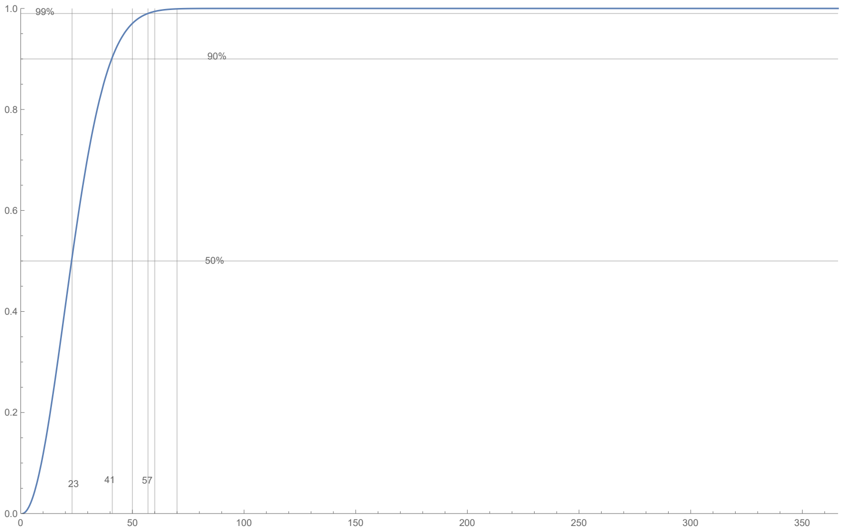
Example

A shipment of 40 *precision parts, including* 8 *that are defective, is sent to an assembly plant. The quality control division selects* 10 *at random for testing and rejects the entire shipment if* 1 *or more in the sample are found to be defective. What is the probability that the shipment will be rejected?*

Birthday Paradox!

Example

Let's assume there are 365 days in a year (sorry anyone born on February 29th). In a group of n people, what is the probability that at least 2 people have the same birthday? What value of n is required for the probability to be at least 50%? 90%? 99%?


1.0
0.8
0.6
0.4
0.2
0.0
99%
90%
50%
23
41
57
0
50
100
150
200
250
300
350